Laser Assisted Cherenkov Emission in Resonant Media

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We theoretically examine the behaviour of Cherenkov radiation in a lossy, dispersive and resonant medium when emission is assisted by an external electromagnetic field. Under the appropriate coherence conditions for Cherenkov emission, we anticipate a large increase of the emission yield at resonance. Our predictions are implemented by numerical estimates for cuprous oxide (Cu_2O).
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A charged particle moving above a critical velocity in a transparent medium emits an unusual type of radiation which was first observed in liquid media by Cherenkov [1] and Vavilov [2] and later interpreted by Frank and Tamm [3]. The study of such an effect was further extended to other condensed media where the emission of Cherenkov radiation turns out to be a very small fraction of the total energy lost by a fast charged particle [4]. The loss of the incident particle energy is in fact mainly due to the ionization and to the excitation of the polarizable medium, however the small loss associated with the emitted Cherenkov radiation can be singled out from the particle total loss through the medium [4 - 6].

It is the purpose of this work to investigate the effect of an external electromagnetic field on the energy lost by fast electrons in a resonant medium. In particular, we predict that under appropriate coherence conditions, the presence of an external laser field induces qualitative as well as quantitative modifications of the Cherenkov emission.

We start by giving an expression for the electromagnetic field emitted by a fast electron crossing the medium. The energy lost by the electron through a thin layer of thickness L is much smaller than its kinetic energy and we may neglect the small decrease of the electron velocity (Born approximation). The presence of an external laser field amounts to adopt a suitably dressed form of dielectric function, which

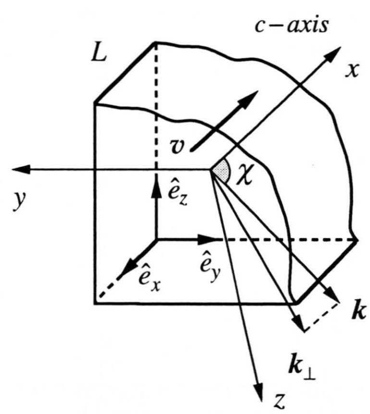

Fig. 1. A fast electron impinges on a thin layer of thickness L that is in turn coherently driven by an external electromagnetic field. The field propagates in the yz plane (principal plane) and is linearly polarized along x (optical c-axis). The electron velocity $\boldsymbol{v}$ is directed along the optical c-axis. The medium principal axes $\hat{x}$, $\hat{y}$ and $\hat{z}$ are oriented respectively along [1,1,1], [1,-1,0] and [1,1-2] with respect to the cristal cubic axes $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$. Upon the passage of the electron, Cherenkov emission occurs in the direction of the wavevector $\boldsymbol{k}$ at an angle χ about the optical c-axis while $\boldsymbol{k}_\perp$ denotes the wavevector projection on the principal plane.

we take here to be *uniaxial* [6], in place of the one that the medium would otherwise have in the absence of the field. The electron is assumed to be a point-charge q_e moving with constant velocity $\boldsymbol{v}$ along a direction that is parallel to the optical c-axis, as shown in Figure 1. Owing to the cylindrical symmetry around the optical axis of the uniaxial medium, it is sufficient to use transverse Fourier expansions for the fields $D(\boldsymbol{r}, t)$, $E(\boldsymbol{r}, t)$, $B(\boldsymbol{r}, t)$ and $H(\boldsymbol{r}, t)$ as well as for the free charge density

$$\rho(\boldsymbol{r}, t) = q_e \delta(x - v_x t)\delta(y)\delta(z), \tag{1}$$

and for the free current density

$$\boldsymbol{J}(\boldsymbol{r}, t) = q_e \boldsymbol{v}\, \delta(x - v_x t)\delta(y)\delta(z). \tag{2}$$

The Maxwell equations [7] for these charge and current densities and a uniaxial dielectric tensor characterized by $\epsilon_{\|}(\omega)$ and $\epsilon_{\perp}(\omega)$ with the optical c axis along x can be solved, and after some lengthy caculation [8] we obtain for the single Fourier component of the electric field inside the slab. (Note that the y and z-components of the radiated field are not required here, as the current has no component in this directions.)

$$E_{\boldsymbol{k}_\perp,\omega}(x)|_x = \frac{q_e}{c^2}\left(\omega + \frac{k_x c^2}{v_x \epsilon_\perp}\right)\frac{4\pi i}{q_\|^2} e^{-ixk_x}, \tag{3}$$

where

$$q_\|^2 = \boldsymbol{k}_\perp^2 - \epsilon_\| \frac{\omega^2}{c^2} + k_x^2 \frac{\epsilon_\|}{\epsilon_\perp}. \tag{4}$$

Here $\boldsymbol{k}_\perp = (k_y \hat{y} + k_z \hat{z})$ denotes the transverse component of the emitted wavevector, whereas the frequency dependencies of all dielectric functions are not explicitly shown here. The solution (3) specifically refer to bulk excitations of both longitudinal and transverse electromagnetic waves.

The energy lost by the electron is the total work done [7] by the emitted electric field on the electron moving within the layer of volume V_{layer} and thickness L, i.e.,

$$W = \int_{V_{\text{layer}}} \mathrm{d}\boldsymbol{r} \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{Re}[\boldsymbol{J}(\boldsymbol{r}, t)] \cdot \mathrm{Re}[\boldsymbol{E}(\boldsymbol{r}, t)] \tag{5}$$

$$= \frac{q_e}{(2\pi)^3} \mathrm{Re} \int_{\infty}^{\infty} \mathrm{d}\boldsymbol{k}_\perp \int_{-\infty}^{\infty} \mathrm{d}\omega \int_{-L/2}^{L/2} \mathrm{d}x\, e^{ik_x x} \boldsymbol{E}_{\boldsymbol{k}_\perp,\omega}(x)|_x,$$

where $k_x = -\omega/v$. Upon inserting (3) into (5) the relevant differential loss becomes [8]

$$\frac{\mathrm{d}W}{\mathrm{d}\omega\, \mathrm{d}\boldsymbol{k}_\perp} = \frac{q_e^2 \omega L}{2\pi^2 v^2} \mathrm{Im}\left\{\frac{1}{\epsilon_\perp q_\|^2} - \frac{\beta^2}{q_\|^2}\right\}, \tag{6}$$

where as usual $\beta = v/c$. By further expressing k_x in terms of the angle χ between the optical c-axis and the wavevector $\boldsymbol{k}$ of the excited mode (in uniaxial materials there are *ordinary* and *extraordinary* modes of propagation [6]; only extraordinary modes, however, can be excited by fast electrons moving parallel to the optical axis)

$$k_x = k\cos\chi = -\frac{\omega}{v}, \tag{7}$$

and by introducing the reduced dielectric tensor

$$\epsilon_r = \frac{\epsilon_\|}{\epsilon_\perp} - 1, \tag{8}$$

the differential energy loss (6) becomes with the help of (4)

$$\frac{\mathrm{d}W}{\mathrm{d}\omega\, \mathrm{d}\boldsymbol{k}_\perp} = -\frac{q_e^2 L \beta^2 \cos^2\chi}{2\pi^2 \omega} \cdot \mathrm{Im}\left\{\frac{1 - (\beta^2 \epsilon_\perp)^{-1}}{1 - \cos^2\chi\,(\beta^2 \epsilon_\| - \epsilon_r)}\right\}. \tag{9}$$

Maxima of this energy-loss function occur at the poles of the imaginary part of (9). In particular, the poles given by the vanishing of the denominator of the imaginary part of (9) correspond to the emission of Cherenkov radiation with a coherence condition given by

$$\cos^2\chi = \frac{\mathrm{Re}[\mathcal{E}] + i\,\mathrm{Im}[\mathcal{E}]}{|\mathcal{E}|^2} \quad (\mathcal{E} \equiv \beta^2 \epsilon_\| - \epsilon_r). \tag{10}$$

For frequencies around the medium resonance the imaginary part of the dielectric function $\mathcal{E}$ is typically smaller than its real part in the absence of the external laser field (weak absorption) while for cuprous oxide (Cu_2O), which we consider below, the difference between the real and imaginary parts becomes even larger when the laser field is on [9]. The condition (10) may then be written as

$$\cos^2\chi \simeq \frac{1}{\mathrm{Re}[\mathcal{E}]}, \tag{11}$$

and real values of χ in (11) occur only for speeds and frequencies such that $\mathrm{Re}[\mathcal{E}] > 1$, which yields a threshold velocity below which Cherenkov emission does not take place, that is

$$\beta^2\,\mathrm{Re}[\epsilon_{\|}] > \mathrm{Re}\Big[\frac{1}{1+\epsilon_r}\Big]\frac{|\epsilon_{\|}|^2}{|\epsilon_{\perp}|^2}. \qquad (12)$$

For non-absorbing uniaxial materials this reduces to the threshold condition,

$$\beta^2\epsilon_{\|} > 1+\epsilon_r, \qquad (13)$$

first obtained by Ginzburg [10]. This threshold depends directly on the relative dielectric function ϵ_r which characterizes in turn departures from the threshold velocity for isotropic media whereby (13) reduces to the known results $\beta^2\,\mathrm{Re}[\epsilon] > 1$ or $\beta^2\epsilon > 1$ for an absorbing [4] or a transparent medium [6], respectively.

It is here worth noting that for very small velocities ($\beta \to 0$) losses due to Cherenkov emission are absent and (9) reduces to the non-relativistic energy-loss function for uniaxial [11] media, i. e.,

$$\frac{\mathrm{d}\,W_{\mathrm{NR}}}{\mathrm{d}\,\omega\,\mathrm{d}\boldsymbol{k}_{\perp}} = \frac{q_e^2 L\cos^2\chi}{2\pi^2\omega}\,\mathrm{Im}\Big\{\frac{1}{\epsilon_{\perp}\left(1+\epsilon_r\cos^2\chi\right)}\Big\}, \qquad (14)$$

In particular, the interpretation of early experiments on graphite anisotropies [12] was just based [13] on this result. Like in (9), maxima occur at the poles of the imaginary part in (14) and correspond to plasmon excitations of the dielectric medium.

The above analysis enables one to characterize the emission of Cherenkov radiation in correspondence to the maxima of (9) and subject to the conditions (11) and (12). This radiation, however, may be heavily damped in a medium with nearly resonant absorption lines, and we here discuss the possibility of effectively increasing the resonant emission of Cherenkov radiation by applying a suitable external laser beam. We consider as an example the case of cuprous oxide (Cu_2O) which exhibits "yellow" exciton resonances and which is a prototype material for the study of excitonic transitions in semiconductors [9, 14]. The dressed form of the dielectric function in the presence of the external beam depends not only on the external field polarization, but also on the detailed structure of the exciton levels involved and turns out to be in general anisotropic [8]. For the sake of simplicity, we here restrict ourselves to the case in which the external field frequency is tuned to the 1S-2P exciton

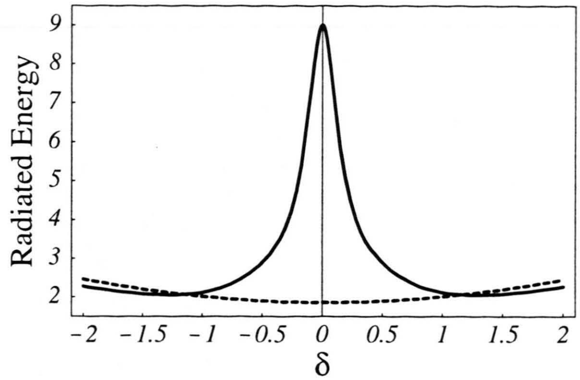

Fig. 2. Differential energy loss (9) per unit length expressed in units of (q_e^2/ω_{2P}) vs. the emitted light frequency in the presence (solid) and in the absence (dashs) of an external driving field. The impinging electron velocity is $v = 0.9c$ while the curves are given at the Cherenkov cone angle $\chi_0 \simeq 64°$. In the presence (absence) of the field the medium becomes uniaxial (isotropic) with the dielecric tensor given by (15) with $A \simeq 0.02$, $\epsilon_\infty \simeq 6.5 + i\,2\times10^{-3}$ and $\gamma_{1S} = 0.1\times\gamma_{2P}$. The emitted frequency, in units of γ_{2P} ($\hbar\gamma_{2P}$ = 1 meV), is measured with respect to the 2P exciton transition frequency ω_{2P} ($\hbar\omega_{2P}$ = 2147 meV). The Rabi frequency of the field is $\Omega_c/\gamma_{2P} = 2$ corresponding to an electric field amplitude of E_c = 30 kV/cm (solid).

frequency splitting [10] while its polarization is along the main cubic diagonal [1,1,1] as shown in Figure 1. For the set of principal axes $\hat{x}$ along [1,1,1], $\hat{y}$ along [1,-1,0] and $\hat{z}$ along [1,1,-2], the resulting dielectric tensor is uniaxial with the optical c-axis parallel to $\hat{x}$ and hence $\epsilon_{xx}(\omega) = \epsilon_{\|}(\omega)$, $\epsilon_{yy}(\omega) = \epsilon_{zz}(\omega) = \epsilon_{\perp}(\omega)$ and $\epsilon_{j\neq k}(\omega) = 0$. Assuming that the three Γ_4^- optically active 2P exciton states are well separated from the other ones of the 2P manifold, the uniaxial tensor components $\epsilon_{\|}$ and $\epsilon_{\perp}$ appropriate for this configuration are

$$\epsilon_{\perp}(\omega) = \epsilon_\infty + \frac{A\,\gamma_{2P}\,(\delta - i\gamma_{1S})}{(\delta - i\gamma_{2P})(\delta - i\gamma_{1S}) - \Omega_c^2/4}, \qquad (15)$$

while $\epsilon_{\|}(\omega)$ is obtained by replacing $\Omega_c \to 2\,\Omega_c$ in the above equation. The Raby frequency Ω_c is proportional to the product of the light beam electric field and the dipole matrix element for the 1S-2P transition [9, 14]. The tensor frequency dependencies are here conveniently expressed in terms of the frequency detuning $\delta = \omega_{2P} - \omega$ of the radiated Cherenkov energy measured with respect to the 2P exciton transition frequency ω_{2P}. Also, A is a numerical constant proportional to the 2P exciton oscillator strength [14],

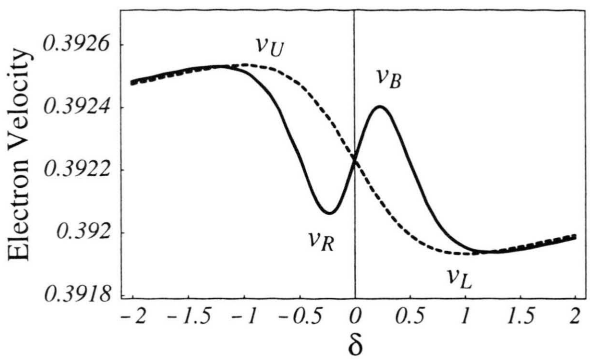

Fig. 3. Threshold condition (11) for the emission of Cherenkov radiation in coherently driven cuprous oxide (Cu_2O). In the resonance region ($|\delta| \leq \gamma_{2P}$) radiation of a given frequency is emitted only for velocities lying above the curves. The electron velocity, in units of c, exhibits the two thresholds v_U and v_L in the absence of the external field (dash) and v_R and v_B in the presence of the field (solid). Notation and field parameters are the same as in Figure 2.

ϵ_∞ is the background dielectric constant, while γ_{1S} and γ_{2P} are respectively the 1S exciton broadening and the 2P exciton linewidth [9, 14]. Typical values for these parameters are given in Figure 2.

For the specific configuration given in Fig. 1 the substitution of (15) and (8) into (11) and (9) gives the coherence condition for the emission of Cherenkov radiation and the corresponding differential energy loss per unit solid angle and frequency. Figure 2 shows indeed the differential loss (9) per unit length in resonance region of the medium; since poles of the numerator of (9) are absent in this region, loss solely arises from the emission of Cherenkov radiation. Such an energy is radiated mainly about a given angle χ_o for which (9) is maximum. This angle defines the characteristic phase cone of the Cherenkov radiation emitted in the frequency range close to the medium resonance. Figure 3 shows instead the electron threshold velocity in the same spectral region. We first observe that for speeds v exceeding the largest threshold v_U in Fig. 3, emission in the presence of the external beam becomes over four times larger than the value it has when the beam is instead switched off. Secondly, in the presence of the external field an increased emission yield appears now to occur at an appreciably lower velocity threshold (v_R) for the frequency slightly above resonance. The reverse occurs for frequencies emitted slightly below resonance, so that upper and lower velocity thresholds appear to be exchanged with respect to the situation of an isotropic sample (no external field). In this frequency range the difference between the threshold in the absence (v_U) and in the presence (v_R) of the external beam can be as large as 10^5 m/sec. Thus near-resonant emission ($|\delta| \leq \gamma_{2P}$) exhibits a gap for electron velocities in the range between v_R and v_B. The "splitting" of the near resonant profile of the threshold velocity in Fig. 3 originates from the "splitting" of the resonant absorption profile in the presence of the external beam.

In this work we discuss Cherenkov emission in a coherently driven resonant medium. For the specific driving field configuration that we examine here we observe a nearly one order of magnitude increase of the resonant emission yield, which would otherwise be damped in the absence of the external field, as well as a modification of the emission coherence conditions. These two important features are due to the combined effects of weak absorption, strong frequency dispersion and anisotropy all induced by the external beam.

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[1] P. A. Cherenkov, Dokl. Acad. Nauk SSSR **2**, 451 (1934).
[2] S. Vavilov, Dokl. Acad. Nauk SSSR **2**, 457 (1937).
[3] I. Frank and I. Tamm, Dokl. Acad. Nauk SSSR **14**, 107 (1937).
[4] P. Budini, Nuovo Cimento **10**, 237 (1953).
[5] E. Fermi, Phys Rev. **57**, 485 (1940).
[6] L. D. Landau and E. M. Lifschitz, Electrodynamics of continuous media, Chapt. 85, Nauka, Moscow 1982.
[7] J. Jackson, Classical Electrodynamics, 2nd Edition, J. Wiley & and Sons New York 1975.
[8] To be discussed elsewhere.
[9] M. Artoni, G. C. La Rocca, and F. Bassani, Europhys. Lett. **49**, 445 (2000).
[10] J. Jelley, in: Cherenkov Radiation, Sect. 3.3, Pergamon Press, New York 1958.
[11] E. Tosatti and F. Bassani, Nuovo Cimento **B 65**, 161 (1970).
[12] K. Zeppenfeld, Z. Phys. **211**, 391 (1968).
[13] F. Bassani and E. Tosatti, Phys. Lett. **27 A**, 446 (1968); E. Tosatti, Nuovo Cimento **B 63**, 54 (1969).
[14] F. Bassani and G. Pastori Parravicini, in Electronic States and Optical Transitions in Solids, Pergamon Press, Oxford 1975; P. Yu and M. Cardona, "Fundamentals of Semiconductors", Springer, Berlin 1996.